Effect of foundation continuity on free vibration of partially supported piles

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ABSTRACT: The natural response of a partially supported pile is investigated idealizing the soil support as a two parameter model. The computed natural frequencies are compared with the cases where the same problem is solved idealizing the elastic foundation as a Winkler model.

1 INTRODUCTION

Flexural and buckling behaviour of beams on an elastic foundation has been extensively investigated and reported in the literature (Hetenyi 1946, Selvadurai 1979 and Scott, 1981). In many of these studies, the elastic foundation is idealized as a Winkler model for mathematical simplicity. Using this simplified approach, Doyle and Pavlovic (1982), Pavlovic and Wylie (1983) and Eisenberger, et al. (1985) have presented solutions for the natural response of partially supported piles on an elastic foundation. One of the major limitations of the Winkler model is the discrete or discontinuous nature of the foundation medium. To overcome this limitation and still maintain the mathematical simplicity of the Winkler model, two parameter models have been used by researchers in the past to investigate the flexural response of beams and beam columns on elastic foundation (Selvadurai 1979, Scott 1981).

In this study the soil is therefore idealized as a generalized two parameter model. Natural frequencies are presented for a wide range of relative stiffness values to investigate the effect of partial support and foundation continuity.

2 ANALYSIS

Figure 1 shows schematically partially supported pile with both ends simply supported. The elastic foundation is idealized by a two parameter model. Figure 2 shows Filonenko - Borodich and Pasternak two parameter rheological models (Selvadurai, 1979). Alternatively, an equivalent two parameter model can be derived by introducing simplifying assumptions starting with a continuum approach (Vlasov model: Selvadurai 1979).

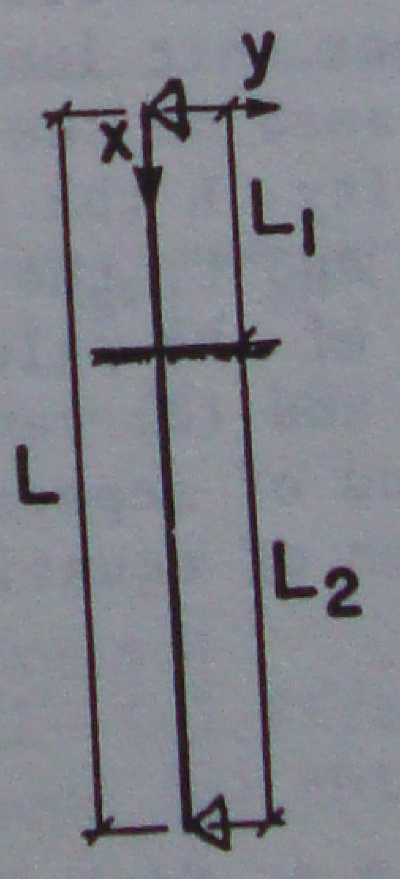

Fig. 1 Partially supported pile with simple supports

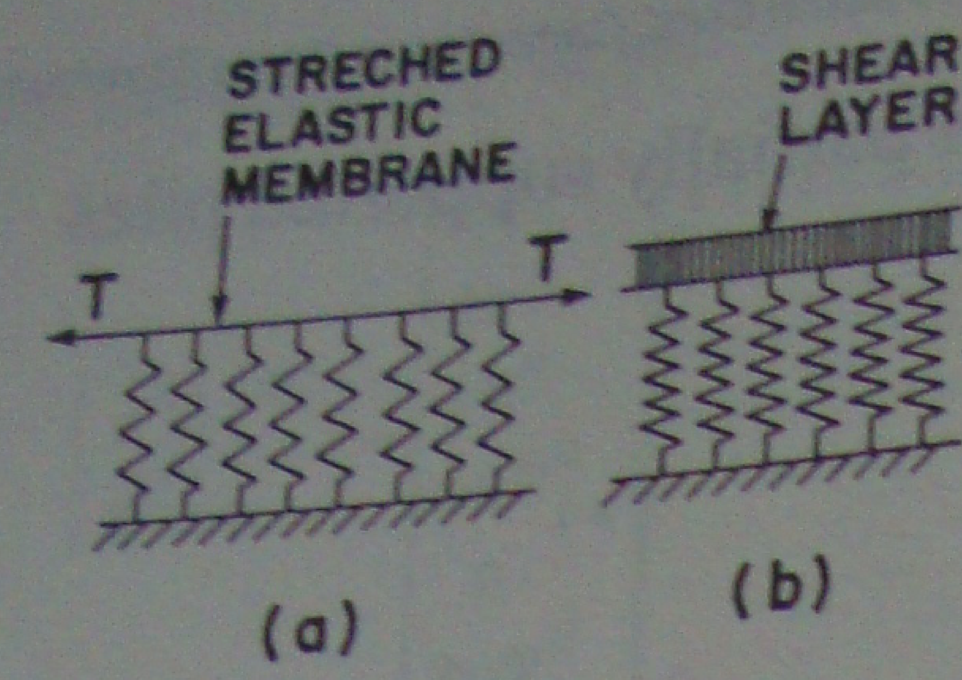

Fig. 2 Two parameter models
(a) Filonenko-Borodich (b) Pasternak

Irrespective of the approach used, the governing partial differential equations for the free vibration of the unsupported and supported portion of the pile are (see Figure 1):

$$EI\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} = 0 \qquad 0 \le x \le L_1 \ldots (1)$$

and

$$EI\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} - S\frac{\partial^2 y}{\partial x^2} + ky = 0 \qquad L_1 \le x \le L. (2)$$

Where y is the lateral displacement; EI the flexural stiffness of the pile, k = kob, where ko is the coefficient of subgrade reaction, b the width of the pile, m the mass per unit length, S the second parameter based on the foundation continuity, L the total length of the pile and L_1 the unsupported length of the pile.

Equations (1) and (2) can be solved by the method of separation of variables. Solution of equation (1) is obtained as:

$$f_1(z) = A_1 \operatorname{Cosh} Cz + A_2 \operatorname{Sinh} Cz + A_3 \operatorname{Cos} Cz + A_4 \operatorname{Sin} Cz \ldots\ldots\ldots\ldots\ldots (3)$$

where $z = x/L$

$C^4 = mL^4\omega^2/EI$, ω = circular natural frequency, and $A_1 \cdots A_4$ are constants of integration.

Equation (2) can be solved by the method of seperation of variables which leads to the following ordinary differential equation:

$$\frac{d^4 f(z)}{dz^4} - \bar{S}\frac{d^2 f(z)}{dz^2} = \eta^4 f(z) \qquad (1-\beta) \le z \le 1 \ldots\ldots\ldots\ldots (4)$$

where $\bar{S} = \frac{SL^2}{EI}$, $\eta^4 = C^4 - \alpha$, $\alpha = \frac{kL^4}{EI}$,

and $\beta = \frac{L-L_1}{L}$

Solution of equation (4) is dependent on whether the value of η^4 is positive, equal to zero, or negative. Solutions for these three cases are as follows:

$\eta^4 > 0$

$$f(z) = B_1 \operatorname{Cosh} \bar{\lambda} z + B_2 \operatorname{Sinh} \bar{\lambda} z + B_3 \cos \lambda z + B_4 \operatorname{Sin} \lambda z \ldots\ldots\ldots (5)$$

where,

$$\bar{\lambda}, \lambda = \frac{\pi}{\sqrt{2}}\{\pm S_R + \sqrt{S_R^2 + (4\eta^4/\pi^4)}\}^{1/2}$$

$S_R = S/\pi^2$ and $B_1 \ldots B_4$ are constants of integration

$\eta^4 = 0$

$$f(z) = B_1 + B_2 z + B_3 \operatorname{Cosh} \pi\sqrt{S_R}\, z + B_4 \operatorname{Sinh} \pi\sqrt{S_R}\, z \ldots\ldots\ldots\ldots\ldots (6)$$

$\eta^4 < 0$

$$f(z) = B_1 \operatorname{Cosh} d_1 z \operatorname{Cos} d_2 z + B_2 \operatorname{Sinh} d_1 z \operatorname{Sin} d_2 z + B_3 \operatorname{Cosh} d_1 z \operatorname{Sin} d_2 z + B_4 \operatorname{Sinh} d_1 z \operatorname{Cos} d_2 z \ldots\ldots\ldots (7)$$

in which $d_2, d_1 = \frac{\pi}{\sqrt{2}}\{\mp S_R + 2\eta^2/\pi^2\}^{1/2}$

Application of appropriate boundary and continuity conditions at the interface between the supported and unsupported portion leads to a set of four homogeneous equations from which the eigenvalues are determined.

3 RESULTS

Numerical results are presented in Table 1 for a fully embedded pile (β=1), and S_R = 0,0.5, 1.0 and 2.5 and α values ranging from 1 to 10^6. For a relative stiffness of α<10, a pile is considered to be rigid; for α in the range of 10 to 1000, a pile is classified as semi-rigid; and for α>1000, the pile is considered to be flexible. Note that S_R=0 corresponds to the Winkler model assumption. Similarly β=0 implies unsupported pile whereas β=1 indicates fully supported or embedded pile.

Table 1. Values of frequency factor "C" for simply supported pile. β=1.0.

α \ S_R	0	0.5	1.0	2.5
FIRST MODE				
1	3.2	3.5	3.8	4.4
10^2	3.8	4.0	4.2	4.6
10^4	10.1	10.1	10.1	10.1
10^6	31.6	31.6	31.6	31.6
SECOND MODE				
1	6.3	6.5	6.6	7.0
10^2	6.4	6.6	6.8	7.2
10^4	10.4	10.5	10.5	10.6
10^6	31.6	31.6	31.6	31.6
THIRD MODE				
1	9.5	9.6	9.7	10.0
10^2	9.5	9.6	9.7	10.0
10^4	11.6	11.6	11.7	11.8
10^6	31.7	31.7	31.7	31.7

The results presented indicate that the natural response of long flexible piles is not affected due to introduction of foundation continuity. However, for rigid and semi-rigid piles introduction of continuity leads to marginally increased natural frequency values for lower modes of vibration.

Due to large number of variables involved in the study and keeping in view the trend observed in Table 1, only typical results are presented for partially supported piles. Results presented are for the case of S_r = 0.5; β=0, 0.25, 0.5, and 1.0; and α values ranging from 1 to 10^6. These particular combinations are considered to cover practical range of parameters encountered in the field situations. The results are presented in graphical form in figure 3.

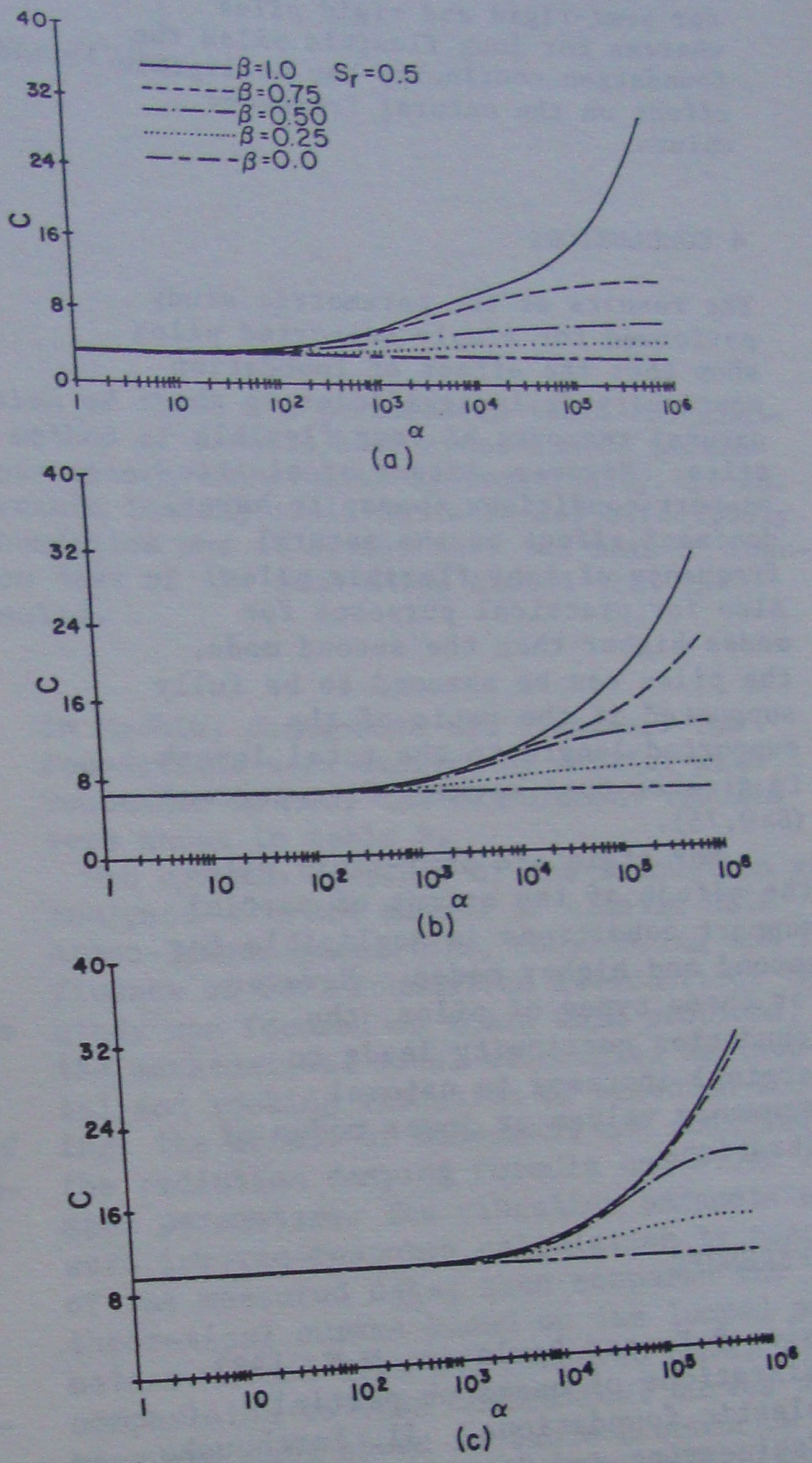

Fig. 3 C versus α for simply supported pile (a) first mode (b) second mode (c) third mode

The results indicate that the natural frequency of rigid and semi-rigid piles vibrating in modes higher than the first mode is not affected by partial soil support conditions. However, the effect of partial support conditions is significant on the natural response of long flexible piles. The introduction of foundation continuity leads to marginal increase in natural frequency values for semi-rigid and rigid piles whereas for long flexible piles the foundation continuity has negligible effect on the natural frequency values.

4 CONCLUSIONS

The results of the parametric study performed for simply supported piles show that the effect of foundation continuity is insignificant on the natural response of long flexible piles. However, extent of elastic support conditions appear to have dominant effect on the natural frequency of long flexible piles. Also for practical purposes for modes higher than the second mode, the piles can be assumed to be fully supported if the ratio of the supported length to the total length is greater than seventy five percent ($\beta \geq 0.75$).

For rigid and semi-rigid piles the effect of the extent of partial support conditions is negligible for second and higher modes. However for these types of piles, the foundation continuity leads to marginal increase in natural frequency values at lower modes of vibrations.

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